0 Additional exercise

a. $\vec{A} \cdot (\vec{B} \wedge \vec{C}) = A_i (\vec{B} \wedge \vec{C})_i$

$= A_i \, \epsilon_{ijk} B_j C_k$ use permutation: $\epsilon_{ijk} = \epsilon_{kij} = \epsilon_{jki}$

order of scalar product is arbitrary

$= A_i \cdot \epsilon_{kij} B_j C_k \overset{\downarrow}{=} C_k \cdot \epsilon_{kij} A_i \cdot B_j$

$= C_k \cdot (\vec{A} \wedge \vec{B})_k = \vec{C} \cdot (\vec{A} \wedge \vec{B})$

$= A_i \cdot \epsilon_{jki} B_j C_k = B_j \, \epsilon_{jki} C_k A_i = B_j \cdot (C \wedge A)_j = \vec{B} \cdot (\vec{C} \wedge \vec{A})$

$= \vec{B} \cdot (\vec{C} \wedge \vec{A})$

b. $\vec{\nabla} \wedge \vec{\nabla}\psi = 0$: $\epsilon_{ijk} \, \partial/\partial x_j \cdot (\partial/\partial x_k \psi) = \partial/\partial x_j \cdot \partial/\partial x_k \psi - \partial/\partial x_k \cdot \partial/\partial x_j \psi$

$\nabla_i = \partial/\partial x_i$

$= 0.$

x_j and x_k are independent - so the order of the partial derivatives does not matter.

c. $[\vec{\nabla} \wedge (\vec{\nabla} \wedge \vec{A})]_i = \epsilon_{ijk} \cdot \partial/\partial x_j \cdot [\vec{\nabla} \wedge \vec{A}]_k$

$[\vec{\nabla} \wedge \vec{A}]_k = \epsilon_{klm} \cdot \partial/\partial_l \cdot A_m$

$\epsilon_{klm} = \epsilon_{mkl} = \epsilon_{lmk}$

$= \epsilon_{ijk} \cdot \epsilon_{lmk} \cdot \partial/\partial x_j \cdot \partial/\partial_l \cdot A_m$

$= [\delta_{il} \, \delta_{jm} - \delta_{im} \cdot \delta_{jl}] \, \partial/\partial x_j \cdot \partial/\partial_l \cdot A_m$

$\delta_{il} \cdot \partial/\partial x_l \, \underbrace{\partial/\partial x_j \, A_m \cdot \delta_{jm}} \; - \; \delta_{jl} \cdot \partial/\partial x_j \cdot \partial/\partial x_l \cdot A_m \cdot \delta_{im}$

$[\vec{\nabla} \cdot \vec{\nabla} \cdot \vec{A}]_i \; - \; [\nabla^2 \vec{A}]_i$